THEORETISCHE PHYSIK IV: STATISTISCHE MECHANIK UND THERMODYNAMIK

Problem Set No. 4

Due on: Friday, 16.5.08 in the practice groups

Exercise 4.1 *(Entropy and Canonical Partition Sum)* **(10 points)**

The entropy of a system with N discrete states is defined as

$$S = -k \sum_{n=1}^{N} p_i \ln p_i.$$

p_i is the probability for finding the system in state i.
Consider now a quantum mechanical system, whose states are (i) a group of g_1 *equally likely* states with energy ϵ_1 and (ii) a group of g_2 *equally likely* states with energy ϵ_2.

(a) Show that the entropy of the system is given by

$$S = -k \left[p_1 \ln \left(\frac{p_1}{g_1} \right) + p_2 \ln \left(\frac{p_2}{g_2} \right) \right]$$

where p_1 and p_2 are, respectively, the probabilities of the system being in a state belonging to group 1 or to group 2 ($p_1 + p_2 = 1$). (2 points)

(b) Assuming that the p_i are given by a canonical distribution, show that

$$S = k \left[\ln g_1 + \ln \left\{ 1 + (g_2/g_1) e^{-x} \right\} + \frac{x}{1 + (g_1/g_2) e^{x}} \right]$$

where $x = (\epsilon_2 - \epsilon_1)/kT > 0$. (4 points)

(c) Verify the foregoing expression for S by deriving it from the partition function of the system. (4 points)

Exercise 4.2 *(Zipper-Model for DNA)* **(10 points)**

A model for the separation of a DNA double strand into its two single strands is the so-called "zipper" model. It is assumed that the double strand can only open up from one single side like a zipper. The zipper has N links. Every link can be either open or closed. The energy associated with an open link is ϵ, the energy associated with a closed link is 0. Each open link furthermore has a degree of degeneracy G denoting the number of possible spatial conformations for an open link. We assume that the zipper can only open up from the left and that for a link to open up, all links on the left hand side of it must be already open. The last (right) link is always closed.

(a) Calculate the canonical partition sum of the system. (4 points)

(b) Find the average number of open links in the limiting case $\frac{\epsilon}{kT} \gg 1$ for a given temperature T. (6 points)

Exercise 4.3 *(Particles in a Magnetic Field)* **(10 points)**

When a particle with spin $\frac{1}{2}$ is inside a magnetic field H, its energy levels are split up in $-\mu H$ and $+\mu H$ with magnetic moment μ or $-\mu$ in the direction of H. Consider a system of N such particles in a magnetic field H at a temperature T. The particles do not interact with each other.

(a) Find the partition sum and the total magnetic moment $\langle M \rangle$ of the system. (5 points)

(b) Determine $\left\langle (M - \langle M \rangle)^2 \right\rangle$. (5 points)